QUIZ 20 SOLUTIONS: LESSON 25
MARCH 25, 2019

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Find the minimum value of $f(x, y) = x^2 + y^2$ subject to the constraint $2y = 5 - x$. Round your answer to 4 decimal places.

$f(x,y) = x^2+y^2$, $g(x,y) = x+2y = 5$

$f_x = 2x$, $f_y = 2y$, $g_x = 1$, $g_y = 2$

System:

$$\boxed{\begin{aligned} 2x &= \lambda \\ 2y &= 2\lambda \\ x+2y &= 5 \end{aligned}}$$

Since $2x = \lambda$, we see $2y = 2\lambda = 2(2x)$

$\Rightarrow 2y = 4x \Rightarrow y = 2x$

Plugging this into the constraint,

$5 = x + 2y = x + 2(2x) = x + 4x = 5x$

$\Rightarrow x = 1$, $y = 2(\underset{x}{1}) = 2$

Our solution is $(x,y) = (1,2)$ and we conclude the minimum value of $f(x,y)$ is

$f(1,2) = 1^2 + 2^2 = 1 + 4 = \boxed{5}$

2. [5 pts] Find the maximum value of $f(x, y) = 30x^{3/2}y$ subject to the constraint $x + y = 58$. Round your answer to the nearest integer.

$f(x,y) = 30x^{3/2}y$ $\qquad$ $g(x,y) = x+y = 58$

$f_x = \frac{3}{2}(30)x^{\frac{3}{2}-1}y$, $\quad f_y = 30x^{3/2}$, $\quad g_x = 1$, $\quad g_y = 1$

$= 45x^{1/2}y$

System:

$$\boxed{\begin{aligned} 45x^{1/2}y &= \lambda \\ 30x^{3/2} &= \lambda \\ x+y &= 58 \end{aligned}}$$

Since both $45x^{1/2}y$ and $30x^{3/2}$ equal λ, we write

$45x^{1/2}y = 30x^{3/2}$

$\Rightarrow 45x^{1/2}y - 30x^{3/2} = 0$ $\quad$ Note: $x^{1/2}x = x^{3/2}$

$\Rightarrow 15x^{1/2}(3y - 2x) = 0$

Hence, either $x = 0$ or $3y = 2x \Rightarrow y = \frac{2}{3}x$.

Case 1: $x = 0$

By the constraint: $58 = 0 + y \Rightarrow y = 58$

$(0, 58)$

Case 2: $y = \frac{2}{3}x$

By the constraint: $58 = x + \frac{2}{3}x = \frac{5}{3}x \Rightarrow x = \frac{3 \cdot 58}{5} = \frac{174}{5}$

And so $y = \frac{2}{3}\left(\frac{174}{5}\right) = \frac{116}{5}$

$\left(\frac{174}{5}, \frac{116}{5}\right)$

$f(0,58) = 30(0)^{3/2}(58)$

$= 0$

$f\left(\frac{174}{5}, \frac{116}{5}\right) = 30\left(\frac{174}{5}\right)^{3/2}\left(\frac{116}{5}\right)$

$\approx \boxed{142882}$

Max